View PDF

Jennifer Lewis has a PhD in teacher education and mathematics education from the University of Michigan, where she teaches classes and conducts research in mathematics education. She also works in Jewish education through her affiliation with the Mandel Foundation. Before that, Jenny taught public school for ten years. She is married to Marc Bernstein and together they have four children. This article appears in issue 7 of Conversations, the journal of the Institute for Jewish Ideas and Ideals.

New ideas about the teaching and learning of mathematics present challenges for Orthodox schools. In part, these ideas about the teaching and learning of mathematics are challenging to *any* schools: teachers lack content knowledge in the subject because they have had insufficient opportunities to learn themselves; teachers are strained pedagogically to teach a subject that they learned differently as students; ambitious aims for subject matter learning compete with a whole host of educational issues that need no enumeration here. For Orthodox schools, new understandings about cognition and learning are particularly fraught. Readers of this journal will not be surprised to read that there are tensions inherent in a stance that embraces *Torah* ***u****Madda*, but in this piece I relate an experience that brought this tension into strong relief for me: conducting a professional development seminar on teaching and learning for heads of modern Orthodox *yeshivot*.

Rabbis and Third Graders Doing Math. To give a glimpse of these tensions, we peek in on a gathering of heads of school and teachers of religious studies from schools that define themselves as modern Orthodox. For this professional development seminar, school leaders from around the United States gathered for three days of collaborative study about teaching and learning.[1] The seminar began with my posing a mathematics problem to the participants, virtually the same problem that they would subsequently watch third graders working on: "I have pennies, nickels, and dimes in my pocket. If I pull out three coins, what amounts of money might I have?" Unaccustomed to doing math problems in a group setting, and even less comfortable making public presentations about their mathematics reasoning, the school leaders shared their solutions to the coin problem and explained how they arrived at their answers. The *rabbanim* came to the chalkboard to show their solutions; they eventually came to consensus that there are 10 possible solutions to the 3-coin problem and collectively constructed an informal proof to convince themselves. The *rabbanim* then turned their attention to the video of third-graders working on a very similar problem that their teacher had posed: "I have pennies, nickels, and dimes in my pocket. If I pull out *two* coins, what amounts of money might I have?"

In the video, we first see the teacher leading the class through a discussion of the parameters of the problem, and the definitions of the terms used. She then sets the students loose to work independently for a few minutes. Children draw or record different possible combinations in their notebooks. Some shuffle coins on their desks to find different arrangements; some draw the coins in their notebooks while others use a range of symbols to show each combination. After working for a while, the teacher asks the children to share their solutions. The discussion proceeds at a slower pace than most mathematics lessons; there are long silences and children offer a number of wrong answers. The teacher gives few comments and little correction; instead, she asks many questions and throws it to the class to determine if a child's answer is correct. She asks repeatedly, "How did you get that?" "How do you know?" "What do other people think about that?"
Here is a brief excerpt from this classroom discussion:

Teacher	Fifteen cents. Could somebody say how they think Sheena made 15 cents. What coins she used to make fifteen cents? Tembe?
Tembe	Ten and a five cent.
Teacher	Okay. Dime ... make a little more room here ... So you had, one nickel and one dime. Okay. Who had another solution besides fifteen cents? What else might I pull out of my pocket? Ofala?
Ofala	Twenty cents.
Teacher	Okay. . How did you get twenty cents, Ofala?
Ofala	Two dimes.
Teacher	Two dimes? Riba, would that work?
Riba	Yes.
Teacher	How do you know?
Riba	Because ten plus ten is twenty.
Teacher	Sean, do you agree with that?
Sean	Huh? Yes.
Teacher	Two dimes would make twenty?
Sean	Yeah.
Teacher	Okay. So we have fifteen cents and twenty cents. Were there any others that you came up with? Tembe, what did you and Devin come up with besides fifteen cents and twenty cents? What's another one you found? What did you guys write down? I know that you found some other ones, I think when I came by. What about this one? How did you get that?
Tembe	That’s his one.
Teacher	Devin, do you remember how you got six cents? You don't remember? Does somebody know how Devin might've gotten six cents? He wrote six cents down in his notebook. How do you think he might've gotten six cents? Betsy?
Betsy	A nickel and a penny?
Teacher	One nickel and one penny. You think that's right, Devin? One nickel and one penny?
Devin	Yeah.
Teacher	Can you show us with your coins? Not in your notebook. Can you get the, can you get a nickel and a penny out of your box? How much is the penny? Okay, the penny is one. And the nickel is ...

Devin	Six cents.
Teacher	Altogether it's six. Good, Devin. Okay. Any others? Mark? Did you come up with any others besides fifteen, twenty and six?
Mark	Eleven.
Teacher	Eleven cents. How did you get eleven cents?
Mark	Ten cents and a penny.
Teacher	One dime and one penny. Did anybody else find that one? Sean, did you come up with eleven cents? Well, what do you think about that? Would that work with a dime and a penny?

Mathematics Teaching and Learning to Teach Project. (1990). Deborah Ball, Third Grade, September 18, 1989 Unpublished transcript. University of Michigan: Ann Arbor, MI. The names of the students have been replaced with pseudonyms.

The assembled *rabbanim* were intrigued by this classroom excerpt. They were keen observers of teaching and learning, despite protests that some had no formal education training. Our seminar used this video and the mathematics work that preceded it as a springboard to discussions of learning and teaching-- in mathematics and in general. In this excerpt, students had reasoned through a complex problem to learn mathematics, and the role of the teacher's authority had shifted from one of providing answers to one of facilitating the reasoning through ideas so that students could come to warranted mathematical conclusions. We saw the teaching of mathematical practices that students could use to develop robust understandings of mathematical ideas. Participants found this image of teaching to be engaging and powerful; a number of them approached me to do continuing work in their schools to develop this kind of teaching and learning school-wide.

I hesitated. Over the days of this professional development seminar, I had become increasingly aware of the tensions between this model of teaching and learning and my understanding of the mandates of Orthodox education. As deeply committed as I am to this kind of teaching and learning, and as much as I want to join with others in the improvement of Jewish education in the Orthodox sector, I am not sure that these two forces are compatible.

In what follows, I will describe how this model has evolved, its antecedents, and why I believe it provides an authentic and rich learning experience in mathematics and in other subjects-- including *limmudei kodesh*. At the same time, I see that the issues that preoccupy even "modern" Orthodox schools today are in some cases orthogonal to this view of learning. It is this tension that I write about in this article.

A "New" View of Teaching and Learning [Mathematics]. Here I elaborate further what is meant by this "model of [mathematics] teaching and learning." I place "mathematics" in brackets because the current wave of educational reform is based on a general view of teaching and learning that extends to mathematics as well as other school subjects.

In the case of mathematics, the model of teaching and learning envisioned goes beyond traditional models where teachers show students how to perform procedures and mathematical routines. Complete understanding...includes the capacity to engage in *the processes* of mathematical thinking, in essence doing what makers and users of mathematics do: framing and solving problems, looking for patterns, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on. Students should not view mathematics as a static, bounded system of facts, concepts, and procedures to be absorbed but, rather, as a dynamic process of "gathering, discovering and creating knowledge in the course of some activity having a purpose." (Stein, M. K., B. W. Grover, and Hennigsen, M., 1996. "Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms." <u>American Educational Research Journal</u> **33**(2): 455-488; emphasis in the original)

Instruction in such classrooms departs in some ways from traditional mathematics instruction. Students reason through problems, and the teacher's authority is less about conferring correctness than it is about helping students learn how to engage in mathematical practices so that they can adjudicate for themselves what is mathematically correct and what is not. This model does not mean that students no longer learn algorithms or have to practice procedures; it also does not mean that each student is free to determine for herself what is correct and what is not-- mathematics instruction will always be directed towards precision, correctness, and convergence around a right answer. Although this model includes these aspects it goes far beyond them as well.

It is clear why this model holds such appeal for the school leaders I worked with. Swap "Torah learning" in place of mathematics above, and most Jewish educators nod their heads in vigorous agreement with this stance towards learning. The image of students engaged in "a dynamic process of 'gathering, discovering and creating knowledge in the course of some activity having a purpose'" is just what school leaders say they want.

This way of teaching mathematics is based in part on a disciplinary view of mathematics. In *Proofs and Refutations* (Lakatos, I., 1981. Proofs and refutations: The logic of mathematical discovery. Cambridge; New York, Cambridge University Press) Lakatos provides an image of how learners arrive at mathematical truths in his description of an imaginary classroom working on a geometry problem respecting the number of vertices and edges and faces in regular polyhedra. (The details of the problem have mostly been omitted for our purposes.)

The dialogue takes place in an imaginary classroom. The class gets interested in a PROBLEM... After much trial and error they notice that for all regular polyhedra V - E + F = 2. Somebody *guesses* that this may apply for any polyhedron whatsoever. Others try to falsify this *conjecture*, try to test it in many different ways-- it holds good. The results *corroborate* the conjecture, and suggest that it could be *proved*. It is at this point-- after the stages *problem* and *conjecture*-- that we enter the classroom. The teacher is just going to offer a proof.

TEACHER: In our last lesson we arrived at a conjecture concerning polyhedra.... We tested it by various methods. But we haven't yet proved it. Has anybody found a proof?...

In Lakatos' description of a classroom, we see his emphasis (in the original text) on the mathematical processes captured in the nouns *guess, conjecture, corroborate,* and *prove.* The classroom dialogue that helps students participate in these practices is a medium in which mathematical conclusions are derived. In a more traditional mathematics classroom, students would be told that V - E + F = 2, and perhaps shown a proof for why this is so. In contrast, in Lakatos' example, students participate in the construction of this proof themselves. This kind of mathematical reasoning is one of the disciplinary images on which current models of mathematics teaching are based. It is centrally concerned with students' deep understanding of the discipline, not just their performance of school tasks.

This model of teaching and learning also draws from wider ideas in the philosophy of education. Israel Scheffler expresses one conceptualization of teaching and learning that underlies this view:

Teaching may be characterized as an activity aimed at achievement of learning, and practiced in such manner as to respect the student's intellectual integrity and capacity for independent judgment. Such a characterization is important for at least two reasons: first, it brings out the intentional nature of teaching, the fact that teaching is a distinctive goal-oriented activity, rather than a distinctively patterned sequence of behavioral steps executed by the teacher. Second, it differentiates the activity of teaching from other activities such as propaganda, conditioning, suggestion, and indoctrination, which are aimed at modifying the person but strive at all costs to avoid a genuine engagement of his judgment on underlying issues. (Scheffler, I.,1965. "Philosophical Models of Teaching." Harvard Educational Review **35**(2): 131-143)

In Scheffler we see where this model of teaching and learning collides with the mandates of an Orthodox education. To what degree, and in what subjects, do our Orthodox schools want to nurture and encourage "independent judgment"? In issues of faith, and in questions of *halakha*, to mention two prominent examples, are we prepared for students to make independent judgments? And these are not tangential subjects in Orthodox schools; one might argue that both issues of faith and questions of practice are the *raison d'etre* for Orthodox schools, and part of what distinguishes them from other streams of schooling. As the seminar with the *rabbanim* progressed, I became more and more aware of the press for their schools to insist on convergence of thought and action in the teaching of particular subjects.

The view of learning depicted here does not apply solely to mathematics. It is not even about a subset of school subjects. It is descriptive-- it *describes* how students learn, generally. This description of how students learn, though, implies a normative view of teaching-- how teachers should teach, given that learning proceeds in this way. And mathematics is perhaps a *kal vahomer* case in the sense that it seems to non-mathematicians as an unlikely discipline to be reasoned through and understood-- and for this reason is even more threatening than perhaps other school subjects. A discipline that was always, at least in the school context, construed as positivist, in which authority for right and wrong was determined by the teacher and the textbook, is instead a discipline --like others-- in which knowledge is socially constructed and the authority for right and wrong is in part determined by what the students reason to be correct, with teacher and textbook guidance. For the Orthodox educator, this has serious implications for how all subjects will be treated. I do not know that the current climate in Orthodox schools can accommodate this stance; on the other hand, teaching that is responsible and responsive to learners requires it.

Challenges of modernity. This small vignette about the teaching and learning of mathematics provides a window onto the challenges of modernity for Orthodoxy. We tend to name the onslaught of media, the vivid intrusion of non-traditional lifestyles into our communities, and constant press of material culture, as major challenges to Orthodoxy. Instead this vignette points to the challenges of epistemologies that recast authority, truth and the creation of knowledge as human constructs. I fully embrace these modernist epistemologies, but do so cognizant and even wary that they do not rest easily with the worldview that has taken hold in the current Orthodox environment. To ignore these new views of learning, in my mind, is to deny how students actually acquire knowledge, habits of mind, and dispositions. This suggests that we will need to imagine educative environments for Orthodox students that, in Scheffler's words, "respect the student's intellectual integrity" and strive for "a genuine engagement of his judgment on underlying issues."

What might such educative environments look like? Here I defer to my colleagues whose primary work is instruction in Orthodox schools, who are engaged with its specifics of context and content on a daily basis, to develop instructional designs particular to this need. I close this article with some broad outlines for the kind of instruction this approach implies in *limmudei kodesh*. First, we would need to imagine the treatment of all *limmudei kodesh* that could be shaped by their disciplinary practices as conducted by experts-- by *talmidei hakhamim*, as we saw in the case of mathematics, such that children would engage in the very practices that more advanced *talmidim* encounter, instead of learning school subjects as "bounded system[s] of facts, concepts, and procedures to be absorbed." One example already present in many schools is the mode of pedagogy found in the traditional *beit midrash* which provides a model of teaching and learning, even for young children. Elie Holzer's analyses of *hevruta* study provide one window into such a practice (See, for example, "What connects good teaching, text study and *hevruta* learning? A conceptual analysis, *Journal of Jewish Education* **72** (3), 2006). To put such practices into play widely, our work in teacher education would be to devise pedagogical scaffolds for teachers so that students can effectively engage in these practices using materials and methods suited to their ages and prior knowledge. It would require, too, revisiting the nature of the teacher's authority in *limmudei kodesh*, one that would acknowledge the wisdom of our sages and teachers and concomitantly put students' thinking at center, bringing both worlds into productive dialogue. We look back to the transcript of a third grade mathematics discussion at the beginning of this article as a model for how such conversations might proceed. A teacher's authority in such environments would be a function of his content knowledge as well as his ability to bring students to engage in the "gathering, discovering and creating knowledge in the course of some activity having a purpose."

But we cannot shy away from such subjects as *dinim* or *halakha,* and the practice of *tefilah*. Here too schools might strive for students' genuine engagement of judgment, to echo Scheffler. Students, even at young ages, would learn to reason through the multiple points of view presented by our sages across the centuries, by the teachers in our schools, and by fellow students. Our schools have tended to teach *dinim* as lists of rules and formulae to memorize, analagous to the $V - E + F = 2$ formula for regular polyhedra. The same can be said for interpretations of *humash*--and in fact most subjects in *limmudei kodesh*. I wonder if we have avoided opportunities for students to reason through ideas rather than memorize them as foregone conclusions, understandably fearful that our children will come to their own conclusions that move them away from Orthodoxy. Instruction in these subjects could be expanded to include the reasoning process of the rabbis, the arguments and stretches of faith that characterize the conversations of *HaZal*. Of course this kind of instruction is already happening in many schools. I want to suggest that this kind of teaching and learning-- even when it comes to *halakha* and questions of faith-- will show a tradition that is robust, multifaceted, and stands up to scrutiny. To address diverse learners-- diverse in *hashkafah,* in family background, in learning styles-- the school curriculum will need to include an array of pedagogical presentations that includes this approach. Rather than threatening our continuity, this pedagogical stance conveys a respect for the individual's intellectual integrity and the ability to reason and come to independent conclusions.

The last decades have seen Orthodox schools overtaken by decidedly non-Modern elements. To recruit knowledgeable teachers who live authentic Jewish lives, Modern Orthodox schools have hired more and more teachers who do not embrace a Modern perspective. This is a pity; our schools need to reflect and generate a particular world-view, and we are missing the opportunity to do so. Our teacher education seminaries need to be guided by a vision of education centered on helping students gain tools to come to warranted conclusions in the intellectual company of one's sages, teachers, and peers. This educational stance could distinguish the contribution of Modern Orthodox to the Jewish education world, and would require the design and scholarship of educational researchers to develop protocols, pedagogical structures, and instructional activities that would carry this vision into practice. Modern Orthodoxy has the capacity for these ambitious goals; our schools and teachers' seminaries can be generative sources for an Orthodoxy where this is the hallmark.

[1] The professional development seminar described here was convened and generously sponsored by the Visions of Jewish Education Project of the Mandel Foundation, Israel. The content presented in the seminar, and the views in this article, are solely the author's.